MODELLING AND SIMULATION OF WATER SYSTEMS BASED ON LOOP EQUATIONS

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Abstract: This paper presents a simulation scheme for water distribution systems based on loop equations. Water networks are large scale and non-linear systems. The operational control of such system has posed difficulties in the past to the human operator that had to take the right decisions, such as pumping more water or closing a valve, within a short period of time and quite frequently in the absence of reliable measurement information such as pressure and flow values. Computer simulations of such systems have alleviated these difficulties. They allowed 'what-if' scenarios to be run by the human operator, giving him the possibility to know in advance the operational problems that can arise in the real-life networks due to malfunctions of valves or burst in pipes. However these computer simulations are generally relying on solving a system of equations and the nodal heads variables have been the most frequently used variables in these computer simulations. More recently the numerical algorithms based on loop equations, also called loop flows algorithms, started to receive more attention due to the reduce size of the matrixes involved in the numerical computations, which relies on the loop structure of the water network rather than the nodes. This can improve the numerical properties of the algorithms that model the real-time behaviour of the network. It is shown that such an efficient simulation scheme can be developed in the context of the loop flows algorithms and some problems that appear from doing this are successfully solved.

Keywords: modeling and simulation, state estimation, network systems, loop flows algorithms.

1. INTRODUCTION

Water network simulation provides a fast and efficient way of predicting the network behavior, calculating pipe flows, velocities, head-losses, pressures and heads, reservoir levels, reservoir inflows and outflows and operating costs.

Simulation of real water distribution systems, that do not consist of a single pipe and cannot be described by a single equation, consists of solving a system of equations. The first systematic approach for solving these equations was developed by Hardy Cross (Cross, 1936). The invention of digital computers allowed powerful numerical techniques to be developed that set up and solve the system of equations describing the hydraulics of the network in matrix form. These numerical methods can be classified as the numerical minimization methods (Contro & Franzetti, 1982), the Hardy-Cross method (Chenoweth & Crawford, 1974), the Newton-Raphson method (Donachie, 1974) and the Linear Theory method (Collins & Johnson, 1975). The last three classes include methods used for the solution of systems of non-linear equations, while the first deals with the search of minimum of a non-linear convex function under linear equality and inequality constraints. Irrespective of the numerical procedure used, the simulation of water networks has led to the development of many methods of network flow analysis using various types of decompositions. Each decomposition expresses the resulting system of equations in terms of a specific type of independent variables: the link flow Q (Wood & Charles, 1972), the loop corrective flows ΔQ_l (Epp & Fowler, 1970; Gofman & Rodeh, 1982), the nodal heads H (Jeppson, 1975), and the mixed node-loop approach (Rossman, 1994). In order to asses the relative merits of the different formulations for solving large pipe network problems, the comparison can be made in terms of simplicity of input, initial solution, size of the system of linear equations and efficiency of solution of the system of equations. The balance of these merits made the combination of the nodal heads and the Newton-Raphson algorithm to be the most frequently used procedure for solving water networks (Rahal, 1980; Powell et al., 1988). Extended time simulations which are used to evaluate system performance over time and allows the human operator to model tanks filing and draining, valves opening and closing, have been implemented based on nodal heads equations (Rao et al. 1978). However the use of nodal equations in network flow analysis disclosed a couple of weaknesses. In (Nielsen, 1989) it has been reported that nodal heads based algorithms have weak convergence for the parts of water network containing low pipe flows.

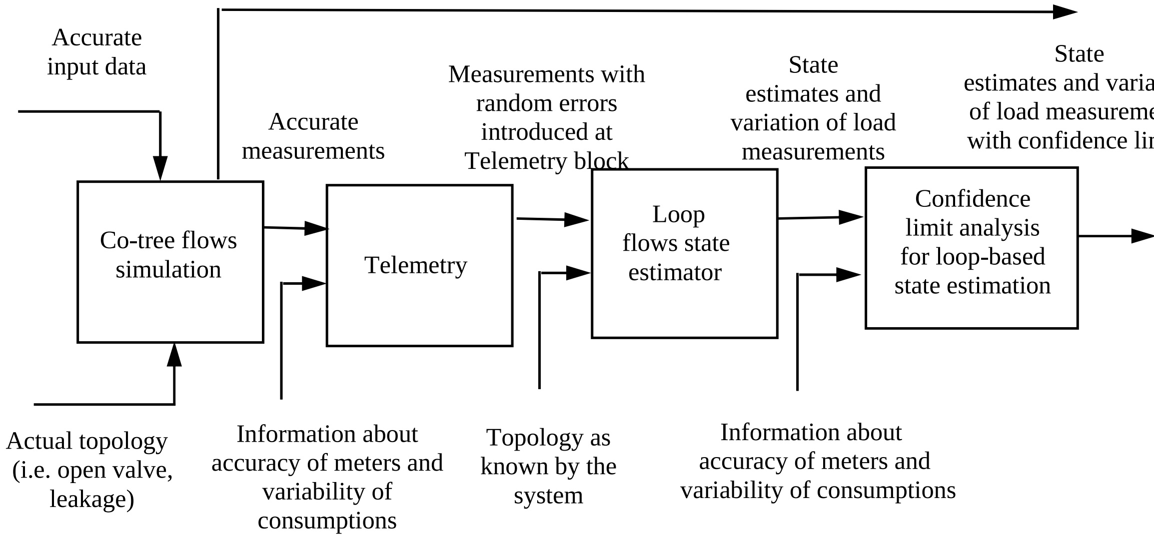

Fig. 1. Operational control of water distribution systems by using loop equations in numerical algorithms.

Finally, the combination of the Newton-Raphson method and the loop corrective flows is called the loop system of equations. Over the last decade the numerical simulations based on loop equations have received an increased attention. It has been shown that the loop equations method is a suitable framework for the inclusion of pressure-controlling elements without specifying the operational state of the network (Andersen & Powell, 1999b). Moreover a rapid convergence has been reported (Andersen & Powell, 1999a; Rahal, 1995) for the simulations of water networks based on the co-tree formulation which is derived from the loop equations method. Although the results were encouraging, no further efforts have been made for developing a fully integrated simulation scheme for on-line monitoring of water networks based on loop equations. This simulation scheme should act as a decision support system that would assist the operational engineer in taking the right decision (e.g. open/close valve) with regard to the status of the distribution system. In this paper we develop such a simulation scheme based on loop flows algorithms.

2. OPERATIONAL DECISION SYSTEM BASED ON LOOP FLOWS ALGORITHMS

The operational control of water systems (e.g. decisions concerning water pumping schedules, pressure control measures, leakage monitoring) is a challenging problem because the models are non-linear and large scale and the measurement information (i.e. pressure and pipe flows or predictions of nodal consumptions) is noisy and frequently incomplete.

In this paper the operational decision system for water networks is based on the block diagram shown at Fig. 1. The figure consists of four simulation blocks: (1) Co-tree flows simulator; (2) Telemetry block; (3) State estimation; and (4) Confidence Limit Analysis (CLA) algorithm. In the following sections, each of the blocks will be presented.

2.1 Co-Tree Flows Simulator Algorithm

A simulator algorithm represents a snapshot in time and is used to determine the operating behaviour of the water network for a set of nodal demands. The simulator algorithm shown here is based on a co-tree flows formulation, which is derived from the loop corrective flows algorithm, defined for a water distribution system with *n*-nodes, *l*-loops, and *p*-pipes.

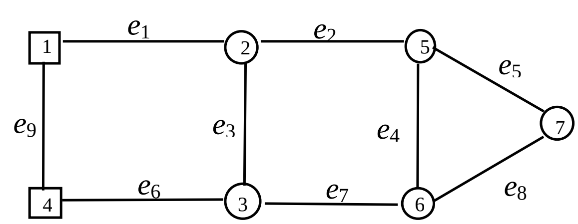

Fig. 2. Water Network from Gofman and Rodeh (1982).

We note with A_{np} (n x p) the topological incidence

System's status (i.e. normal operating state, leakage between node *i* and *j*, etc.)

matrix that has a row for every node and a column for every branch (component) of the network. The non-zero entries for each row +1 and –1 indicate that the flow in pipe *j* enters or leaves node *i*.

$$A_{np}(i,j)=\begin{cases}1 & flow\ of\ pipe\ j\ enters\ node\ i\\ 0 & pipe\ j\ is\ not\ connected\ with\ node\ i\\ -1 & flow\ of\ pipe\ j\ leaves\ node\ i\end{cases} \quad (1)$$

The loop incidence matrix M_{lp} is the (l x p) matrix with the following properties:

$$M_{lp}(j,k)=\begin{cases}1 & flow\ of\ pipe\ k\ flows\ clockwise\ in\ loop\ j\\ 0 & pipe\ k\ does\ not\ pertain\ to\ loop\ j\\ -1 & flow\ of\ pipe\ k\ flows\ anti-clockwise\ in\ loop\ j\end{cases} \quad (2)$$

The co-tree flows simulator algorithm is based on the decomposition of the water network in a spanning tree. A spanning tree for a network (with n nodes and p pipes) is a subnetwork (with n^1 nodes and p^1 pipes) such that $n = n^1$ and the subnetwork contains at least one pipe and no loops, and is said to be *connected*. A network is connected if for every pair of different nodes n_1 and n_2, there is a *path* between them. A path represents a finite sequence of nodes and pipes between the initial node n_1 and the terminal node n_2 and no node or pipe is repeated in the path. The pipes that do not belong to the spanning tree are called *co-tree* pipes or *chords* (e.g. dashed arrows in Fig. 3), and are providing the loop information.

Different search strategies can be employed in order to obtain the spanning tree. In this paper the Depth First (DF) search has been used to obtain the network information (i.e. the spanning tree, the loop and the topological incidence matrixes) (Gofman & Rodeh 1982; Andersen & Powell, 1999b). Concomitant with building the spanning tree, new labels are assigned to pipes and nodes during the search of the water network so that to model the topological incidence matrix as an upper form tree incidence matrix T and a co-tree incidence matrix C (i.e. $A_{np} = [T \quad C]$).

Based on the DF search the topological and the loop incidence matrixes are obtained. The co-tree flows simulator algorithm requires that the initial flows Q_i to respect the mass continuity equation (i.e. the amount of flow that enters to a node equals the amount of flow that leaves the node plus the water consumption in the respective node). This is because the governing system of equations of the co-tree flows simulator algorithm does not account either implicitly or explicitly for continuity.

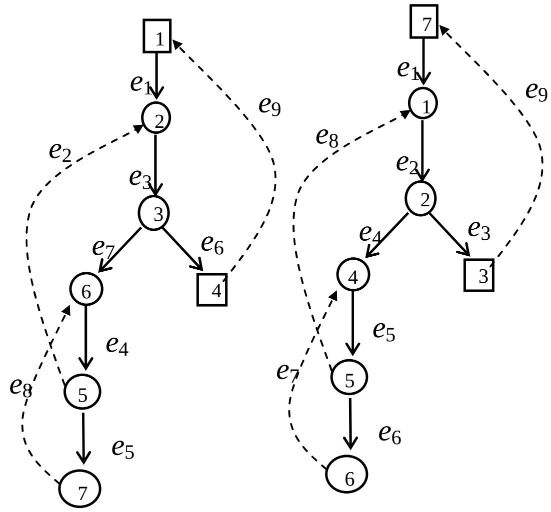

Fig. 3. a) Spanning tree for the water network from Fig. 2, b) New labels for pipes and nodes.

Based on the DF search the topological and the loop incidence matrixes are obtained. The co-tree flows simulator algorithm requires that the initial flows Q_i to respect the mass continuity equation (i.e. the amount of flow that enters to a node equals the amount of flow that leaves the node plus the water consumption in the respective node). This is because the governing system of equations of the co-tree flows simulator algorithm does not account either implicitly or explicitly for continuity. The initial flows Q_{Ti} in tree pipes can be calculated as follows:

$$Q_{Ti} = T^{-1} d \qquad (3)$$

The co-tree flows simulator algorithm consists of the energy equation that has to be satisfied, that is the vector of loop head losses residuals ΔH (l x 1) must be equal to zero:

$$\Delta H=0 \qquad (4)$$

The vector of loop head losses residuals ΔH can be written as:

$$\Delta H=M_{lp}h \qquad (5)$$

where h (p x 1) is the vector of pipe head losses described by the Hazen-Williams equation:

$$h=k\tilde{Q}^n \qquad (6)$$

Here k (p x 1) is the vector of pipe resistance coefficients and $\tilde{Q}$ (p x 1) is the vector of pipe flows that we want to determine. Equation (5) is solved with the Newton-Raphson method which is a well-known numerical method for solving a system of non-linear equations:

$$\Delta Q_{l_{t+1}} = \Delta Q_{l_t} - \left[\frac{\partial \Delta H}{\partial \Delta Q_{l_t}} \right]^{-1} \Delta H \qquad (7)$$

The matrix $\frac{\partial \Delta H}{\partial \Delta Q_{l_t}}$ (l x l) is the Jacobian matrix, that is the derivatives of the loop head losses residuals ΔH with respect to the loop corrective flows ΔQ_{l_t} at the t-th step of the Newton-Raphson iteration process.

For the co-tree flows simulator algorithm the solution is given by vector of pipe flows $\tilde{Q}$:

$$\tilde{Q} = Q_i + M_{pl}\, \Delta Q_l \qquad (8)$$

where M_{pl} (p x l) is the transpose of the loop incidence matrix and ΔQ_l (l x 1) is the vector of loop corrective flows or co-tree flows.

While for the well maintained water distribution systems the normal operating state data can be found in abundance, the instances of abnormal events (e.g. leakages) are not that readily available. In order to observe the effects of abnormal events in the physical system one sometimes is forced to resort to deliberate closing of valves or opening of hydrants (to simulate leakages) (Carpentier & Cohen, 1993). Although such experiments can be very useful to confirm the agreement between the behaviour of the physical system and the mathematical model, it is not feasible to carry out such experiments for all pipes and valves in the system during the whole day or days as might be required in order to obtain the representative set of labelled data.

It is an accepted practice that, for processes where the physical interference is not recommended or even dangerous, mathematical models and computer simulations are used to predict the consequences of some emergencies so that one might be prepared for quick response. In conclusion the co-tree flows simulator algorithm from Fig. 1 is used not only to simulate the normal operational point of a water network but also to predict the consequences of some emergencies so that one might be prepared for quick response.

Furthermore the accurate pressure and flow values obtained from the co-tree flows simulator algorithm are input into the "telemetry" block where random errors, with values defined by the accuracy of meters and the maximum variability of consumptions, are added to accurate measurements to simulate the noisy environment of the real water distribution system. These noisy measurements are sent to the "estimation" block that calculates, for a given measurement set, the state estimates.

2.2 Loop Flows State Estimator

In the operational decision support of water networks, state estimation is an important element that enables processing of diverse measurements obtained via the real-time telemetry systems and facilitates calculation of the best approximation of the operational state of the system.

The state estimation can be viewed as the process of optimisation of a suitably chosen cost function. The least square (LS) criterion, where the sum of the squared differences between the measured and estimated values is minimised, has been intensively used in the operational control of water networks.

The nodal heads equations have been predominantly used as the state variables in the LS state estimators. Although the mathematical model is accurate, it leads sometime to difficulties in modeling and simulation of realistic water networks (Nielsen, 1989; Gabrys & Bargiela, 1996; Sterling & Bargiela, 1984; Powell et al., 1988; Hartley and Bargiela, 1993). An alternative to the nodal heads equations are the loop corrective flows. This can be an advantage because of the smaller size of the matrixes involved in the numerical computations. A Weighted Least Square (WLS) state estimator based on the loop equations and the state variables were the unknown nodal demands was presented in Andersen and Powell (1999a). The minimization problem was solved using a Lagrangian approach. A LS state estimator was shown in Arsene and Bargiela (2001) that employed both the variation of nodal demands and the loop corrective flows as state variables. This estimator is depicted in the following paragraph.

An additional set of variables has been therefore considered, that is the variation of nodal demands Δd. Hence the hydraulic model $g()$ for head, flow and demand measurements becomes a function of both the loop corrective flows ΔQ_l and the variation of nodal demands Δd. The advantage of using the variation of nodal demands is that we are able to write the network equations based on the topological information obtained from the spanning tree.

The pipe flows will be written function of the loop corrective flows ΔQ_l and the variation of nodal demands Δd as follows:

$$\tilde{Q} = Q_i - A^* \Delta d + M_{pl}\, \Delta Q_l \qquad (9)$$

where $\tilde{Q}$ are the pipe flows in tree and co-tree pipes, $Q_i = \begin{bmatrix} Q_{T_i} \\ 0_l \end{bmatrix}$ are the initial flows in tree pipes and co-tree pipes (i.e. zero vector 0_l (l x 1)), and matrix A^* is the matrix with the property $A^* = \begin{bmatrix} T^{-1} \\ 0_{ln} \end{bmatrix}$. There are two sets of equations which will describe the hydraulics of the water network.

The first set of equations states that the loop head losses are equal to zero:

$$\Delta H(\Delta Q_l, \Delta d) = 0 \qquad (10)$$

where the loop head losses residuals ΔH are a function of the loop corrective flows y (ΔQ_l) and the variation of nodal demands x (Δd) and are calculated from equation (5).

The second set of equations states that the total amount of inflow/outflow from the water network carried out through the fixed-head nodes should equal the variation of nodal demands. This equation can be written as follows:

$$\Delta d = B_{nl}\, \Delta Q_l \qquad (11)$$

The incidence matrix B_{nl} (n x l) from equation (11) has a non-zero element equal to 1 which corresponds to the main root node and -1 for each of the fixed-head nodes.

The equations (10) and (11) represents the hydraulic function that describes the water network. It can be written as a system of equations:

$$\begin{cases} \Delta H(\Delta Q_l, \Delta d) = 0 \\ B_{nl}\, \Delta Q_l - \Delta d = 0 \end{cases} \qquad (12)$$

If we denote with $g(\hat{x}, \hat{y})$ the non-linear equation that describes the residuals in the loop head losses and the variation of nodal demands then the variation of the state variables $\begin{bmatrix} \Delta\hat{x} \\ \Delta\hat{y} \end{bmatrix}$ (loop corrective flows and variation of nodal demands) during the Newton-Raphson method is calculated as:

$$\begin{bmatrix} \Delta\hat{x} \\ \Delta\hat{y} \end{bmatrix} = (J^T J)^{-1} J^T\, g(\hat{x}, \hat{y}) \qquad (13)$$

The matrix J represents the Jacobian matrix (i.e. first derivative of function g) and it has been presented in Arsene and Barigela (2001):

$$J = \begin{bmatrix} \frac{\partial \Delta H}{\partial \hat{x}} & \frac{\partial \Delta H}{\partial \hat{y}} \\ -I_{nn} & 0_{nl} \end{bmatrix} \qquad (14)$$

where I_{nn} is the square identity matrix of size n.

Now the LS estimate of $\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}$ can be found with an iterative process with the consecutive state estimates calculated with the following equation:

$$\begin{bmatrix} \hat{x}^{(k+1)} \\ \hat{y}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \hat{x}^{(k)} \\ \hat{y}^{(k)} \end{bmatrix} + \begin{bmatrix} \Delta\hat{x}^{(k)} \\ \Delta\hat{y}^{(k)} \end{bmatrix} \qquad (15)$$

If all elements of $\begin{bmatrix} \Delta\hat{x}^{(k)} \\ \Delta\hat{y}^{(k)} \end{bmatrix}$ at k-th step of the estimation process are lower or equal to a predefined convergence accuracy, the iteration procedure stops. Otherwise, a new correction vector is calculated using equation (15) with $\begin{bmatrix} \hat{x}^{(k+1)} \\ \hat{y}^{(k+1)} \end{bmatrix}$ instead of $\begin{bmatrix} \hat{x}^{(k)} \\ \hat{y}^{(k)} \end{bmatrix}$.

Both the co-tree flows simulator and the loop flows state estimator have been tested on real water networks and the numerical results have shown good convergence (Arsene & Barigela, 2001; Bargiela, Arsene & Tanaka, 2002).

2.3 Confidence limit analysis - Loop Flows Approach

The measurement uncertainty has an impact on the accuracy with which the state estimates are calculated. It is important, therefore, that the system operators are given not only the values of flows and pressures in the network at any instant of time but also that they have some indications of how reliable these values are. The procedure for the quantification of the inaccuracy of the state estimates caused by the input data uncertainty was developed in the late 1980s and termed Confidence Limit Analysis (CLA) (Bargiela & Hainsworth, 1989). Rather than a single deterministic state estimate, the CLA enables the calculation of a set of all feasible states corresponding to a given level of measurement uncertainty. The set is presented in the form of upper and lower bounds for individual variables and hence provide limits on the potential error of each variable (Gabrys & Bargiela, 1996).

This section addresses the problem of CLA based on the loop flows state estimator and the co-tree flows simulator algorithm shown in the previous paragraphs.

In normal use, deterministic state estimators produce one set of state variables for one measurement vector. Used in this way, they give no indication of how the state variables may be affected by the fuzziness of input data. Alternatively, if a deterministic state estimator is used repeatedly for each measurement modified with its defined maximum variability, then a matrix S^e can be determined as:

$$S^e = \frac{\Delta x_i}{\Delta z_j} \quad i=1,\dots n;\ j=1,\dots m \qquad (16)$$

where $i=1,\dots n$ is the index for the state vector denoted with x^t (nodal heads and in/out flows) and $j=1,\dots m$ is the index for the measurement vector denoted with z^t. The measurement vector z^t comprises the estimates for the water consumptions and the fixed-head nodes. It can be augmented with real pressure and flow meters.

The loop flows state estimator has been the deterministic state estimator used to obtain matrix S^e. Matrix S^e is called the Experimental Sensitivity Matrix (ESM) because is obtained through a number of successive simulations. It expresses the variation, Δx, of the *i*-th element, x_i, of the true state vector, x^t, because of a perturbation, Δz, in the *j*-th element, z_j, of the true measurement vector z^t. The true state of the system is not known but instead the best state vector available $\hat{x}$ is used in the process of determining the sensitivity matrix and the confidence limits.

Having found matrix S^e, we can carry out the maximization process expressed by equation (17) in order to obtain the confidence limits for the state variables. For the *i*-th state variable, calculating its error bound x_{cli} is done by maximizing the product between the *i*-th row of the experimental sensitivity matrix s_i and the vector Δz. The maximization process is performed separately for each row of the sensitivity matrix determined in the previous section:

$$x_{cli} = max\ s_i \Delta z \qquad (17)$$

The method presented here gives comparable results with the confidence limits analysis algorithms developed in the context of the nodal heads state estimator (Arsene, 2004).

3. NUMERICAL RESULTS

While for the well maintained water distribution systems the normal operating state data can be found in abundance, extended time simulations (i.e. simulations that stretch over longer periods of time) or instances of abnormal events are not that readily available. In order to observe the effects of abnormal events in the physical system one sometimes is forced to resort to deliberate closing of valves or opening of hydrants (to simulate leakages) (Carpentier & Cohen, 1993). Although such experiments can be very useful to confirm the agreement between the behaviour of the physical system and the mathematical model, it is not feasible to carry out such experiments for all pipes and valves in the system during the whole day or days.

It is an accepted practice that, for processes where the physical interference is not recommended or even dangerous, mathematical models and computer simulations are used to predict the consequences of some emergencies so that one might be prepared for quick response. The block diagram depicted at Fig. 1 was used to generate data covering 24 hour period for the water network shown at Fig. 4. The reason for choosing this network is that we were able to compare the numerical results obtained in the context of loop algorithms with the numerical results presented in Gabrys (1997) for the same water network and operational testing conditions in the context of the nodal heads equations.

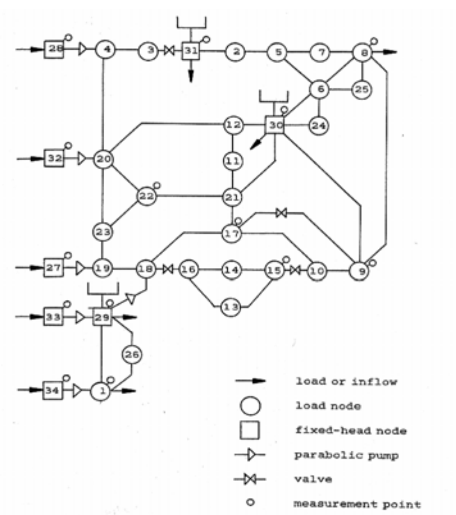

Fig. 4. 34 - node water network.

3.1 Extended Time Simulations

The process of generating the operational data was shown in the form of block diagram at Fig 1. It consisted of three major blocks. The first module was the co-tree flows simulator that could be used as a substitute for the physical water distribution system. It is this module where the leakages are simulated by updating the topology information rather than opening hydrants. In the second module, the loop flows state estimation process is carried out for accurate measurements taken from the simulation module but without knowledge of any anomalous event that might have happened, as would be the case in the real distribution network. In the third module the confidence limits are found for state estimates and the variation of nodal demands calculated at the estimation stage. Additionally to the state estimates with their confidence limits the system's status or label of the current pattern is stored.

However, the algorithms presented in this paper have been developed as standalone applications that are not communicating each other, as would be the case in an extended time simulation. It is well known that at each step in the series of extended time simulations, a new set of nodal demands d^{new} and head values at the boundary nodes of the network are usually provided. In the context of the loop flows algorithms this would require re-calculation of the initial pipe flows, the loop and the topological incidence matrixes. This would further imply that we would have to carry out at each step in the extended time simulation the time consuming process of rebuilding the spanning tree. This is unfavourable if we compare it with the implementation based on the nodal heads equations (Gabrys, 1997) which does not require the re-calculation of the loop and the topological incidence matrixes. However, by simple matrix operations we can avoid this drawback.

Let us assume that M_{lp}, Q_i and T are the loop incidence matrix, the initial pipe flows and the tree incidence matrix obtained from the spanning tree as described at section 2.1. The nodal demands d are given. The simulation block diagram is pursued once. The state estimates with the confidence limits and the status of the water network are stored for subsequent utilization in the process of decision making. For the following step in the series of extended time simulations, a new set of nodal demands is given. The initial conditions (i.e. initial pipe flows, incidence matrixes) are determined with the equation:

$$Q_{Ti}^{new} = T^{-1} d^{new} \qquad (18)$$

where Q_{Ti}^{new} are the initial tree pipe flows re-actualized at each step in the extended time simulation. Furthermore the loop and the tree incidence matrixes are calculated based on the direction of the new derived tree pipe flows. If the direction of initial flows Q_{Ti}^{new} changed when compared with Q_{Ti} then the loop and the tree incidence matrixes are updated as follows:

$$M_{lp}^{new}(1:l,k) = (-1)\ M_{lp}(1:l,k) \qquad (19)$$

$$T^{new}(1:n,k) = (-1)\ T(1:n,k) \qquad (20)$$

where k is the pipe with the reversed flow and M_{lp}^{new} and T^{new} are the new loop and tree incidence matrixes used at the next step of the extended time simulation.

The block diagram shown at Fig. 1 has been successfully run for a 24 hours extended time simulation. The Central Processing Unit (CPU) times were similar with the times obtained for the implementation of the same block diagram based on the nodal heads equations (Gabrys & Bargiela, 1996; Gabrys, 1997). The 24 hour profiles of consumptions and inflows that characterize the normal operating states throughout the day were similar with the ones reported in Gabrys (1997) and are shown below for completeness.

In Fig. 5 are shown the patterns of water consumption at nodes 1, 8, 29, 30 and 31 for a 24 hours extended time simulation. The consumptions are expressed in meters cube per second. In Fig. 6 are shown the patterns of inflows at the fixed head nodes 27, 28 and 32 expressed also in meters cube per second. It is shown also the flow in the pump between nodes 29 and 18. In Fig. 7 are shown the 24 hours profile of inflows at the fixed head nodes 33, 34 in meters cube per second and the heads in meters at reservoir nodes 29, 30 and 31.

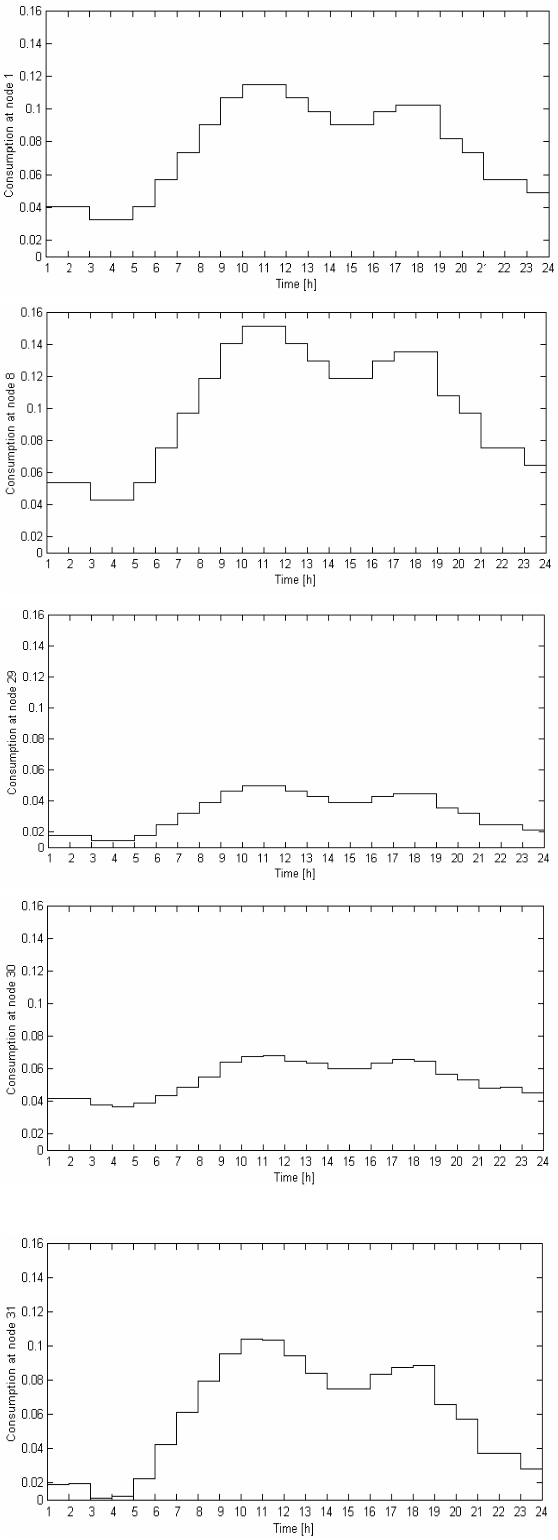

Fig. 5. 24 hour profiles of consumptions at nodes 1, 8, 29, 30 and 31.

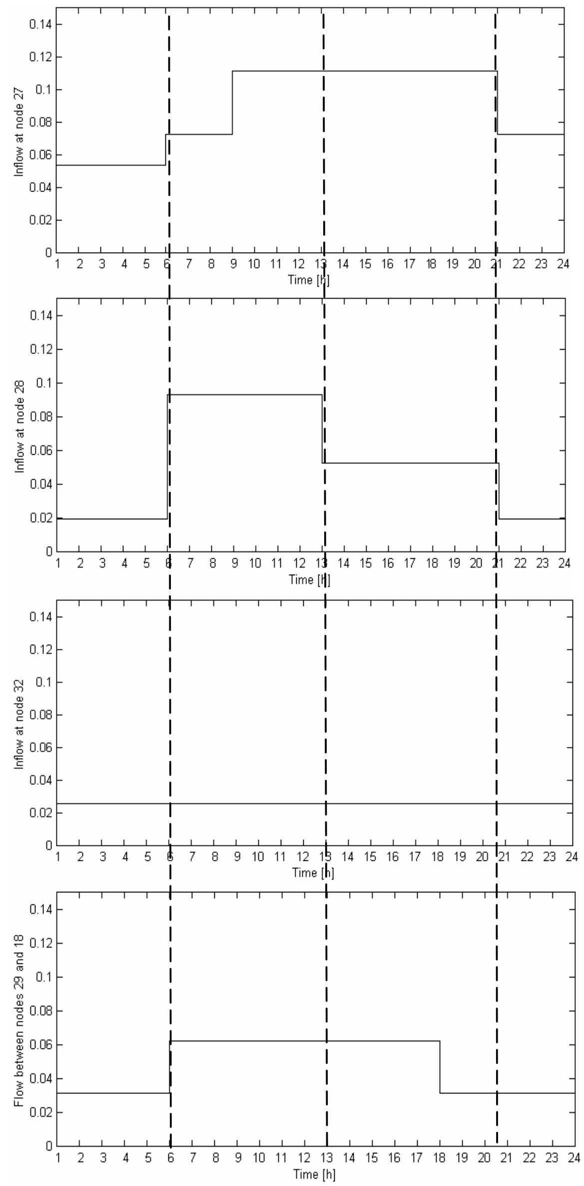

Fig. 6. 24 hour profiles of inflows at fixed head nodes 27, 28, 32 and booster pump between nodes 29 and 18.

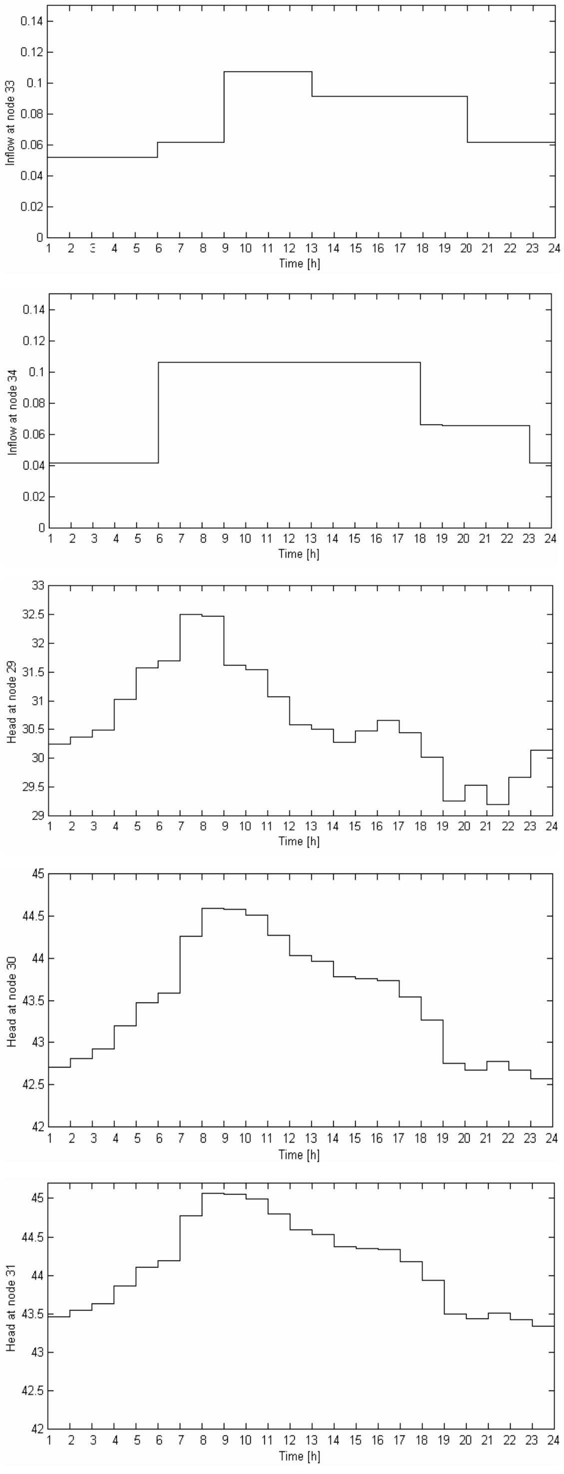

Fig. 7. 24 hour profiles of inflows at fixed head nodes 33, 34 and heads at reservoir nodes 29, 30, 31.

3.2 Simulation of Leakages

The leakages are modeled as an additional demand lying midway between the two end nodes of a pipe during the co-tree flows simulation of the water network. The additional demand is not modeled as a pressure dependent variable and thus could be set to any desired value. By systematically working through the network, ten levels of leaks are introduced, one at a time, in every single pipe for every hour of the 24 hour period. Since there are 38 pipes multiplied by 10 levels of leakages and plus the normal operating status gives 381 patterns of state estimates and confidence limits for each hour. For a full day this produces a set of experimental data of 9144 patterns of state estimates and confidence limits computed for measurements and leakages ranging from 0.002 to 0.029 [m^3/s].

Since an additional consumption is used in order to simulate the leaks, this would require recalculating the incidence matrixes and the initial pipe flows. Rebuilding the spanning tree for each of the 9144 patterns of data would represent a computational drawback for the simulation scheme shown at Fig.1 and a disadvantage when compared to the implementation based on the nodal heads equations, which once again does not require the recalculation of the incidence matrixes and the initial pipe flows.

An extremely simple and efficient solution is adopted here. We use graph operations in order to modify the spanning tree (Fig. 8) which corresponds to the normal operating state of the water network. This way we take into account the additional water consumption that models the leak. Furthermore, the new incidence matrixes and initial flows are determined for the simulator algorithm which avoids the time consuming process of rebuilding the spanning tree.

Let us introduce a leakage in the pipe between nodes 17 and 18. In order to simulate the 35-node water network (i.e. the original 34-node water network plus the leakage modeled as an additional consumption) the incidence matrixes and the initial flows are recalculated. The labels for nodes and pipes situated in the spanning tree below the leakage location, are incremented by one so that to preserve the upper form of the tree incidence matrix. The reader should observed that for the part of the network delimited by nodes 26, 29, 33, and 24 no leaks will be considered because it is separated from the rest of the network by a pump with constant flow.

The vector of nodal demands $d^{''}$ comprises the initial nodal demands d plus the leakage that is introduced as a distinct element in the vector of water consumptions. A column and a row are introduced in the incidence matrixes (loop and topological) so that to take into account the incidence of the two half-pipes resulted from the additional demand. Following this, the initial pipe flows and the loop and the tree incidence matrixes are obtained through simple matrix operations (equations (18), (19) and (20)) more efficient to use in terms of computational time than to reconstruct the spanning tree for the 36–node water network.

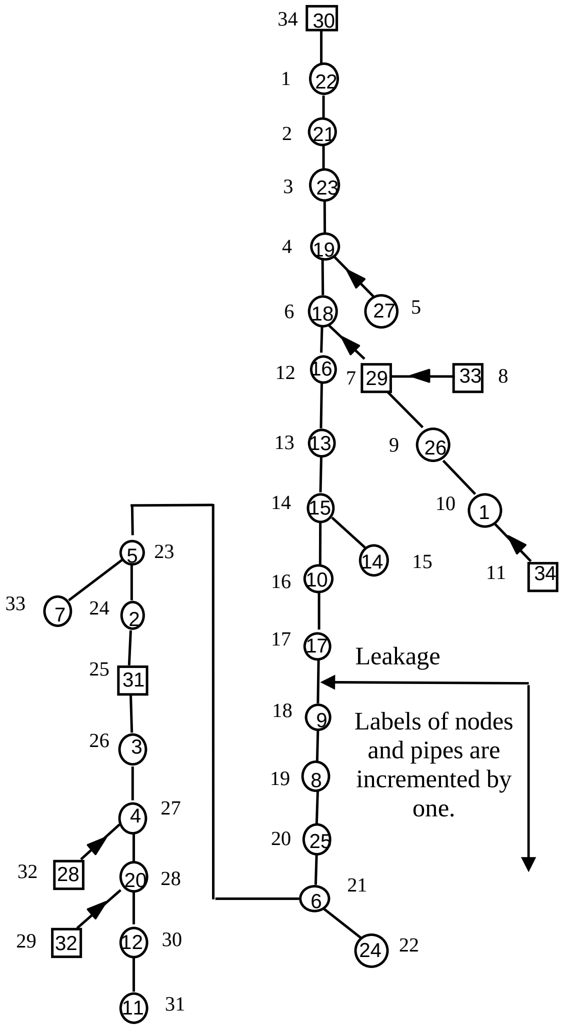

Fig. 8. Graphical representation of the spanning tree for the 34-node water network.

The total CPU time for generating the 9144 set of data based on re-building the spanning tree for each of the 9144 patterns of data was 15 minutes. However by using graph and simple matrix operations we were able to reduce the computational time to 40 seconds which is comparable with the CPU times obtained with the nodal heads equations.

Head measurements	1, 2, 4, 8, 11, 15, 17, 19, 22, 29, 30, 31
Fixed-head inflow measurements	27, 28, 29, 30, 31, 32, 33, 34
Water consumptions	All nodes
Fixed-head measurements	27, 28, 29, 30, 31, 32, 33, 34
Leak levels	0.002, 0.005, 0.008, 0.011, 0.014, 0.017, 0.020, 0.023, 0.026, 0.029 [m^3/s]
Parameters used in confidence limit analysis	
Accuracy of head measurements at load nodes	+-0.1[m]
Accuracy of inflow measurements	+-1%
Variability of consumptions	+-10%

Table 1. Parameters used during generation of the 9144 numerical data set.

3. CONCLUSIONS

The purpose of this paper was to investigate the implications of the loop equations formulation of the simulator algorithm, state estimation procedure and confidence limit analysis for the implementation of decision support systems in the operational control of water networks. The nonlinear models and large scale of the water distribution systems made them both a challenging problem to be tackled and a very good validation example for a prototype decision support system useful in other utility systems.

The paper has been divided in two distinctive parts. In the first part, we used the loop equations for the implementation of a co-tree flows simulator algorithm, we presented a novel loop flows state estimator, and a confidence limits analysis algorithm based on the loop flows variables. A particular emphasis has been placed on the fast calculation of the initial input data (the incidence matrixes and the initial pipe flows), enhancement and accuracy of the results and good convergence properties for the numerical algorithms.

The second part of the paper was concerned with combining the numerical algorithms in an efficient simulation block scheme for a 34-node water network. It has been shown that instead of carrying out the time consuming process of re-building the spanning tree for each simulation in the series of extended time simulations or for leakage simulations, graph and matrix operations can be used in order to drastically reduce the CPU times.

We can conclude that all the developed modules have been successfully integrated into an efficient simulation block diagram that has been used to generate numerical data for a realistic 34-node water network without having to actually carry out experiments for pipes and valves in the real-life system during the whole day or days. These numerical data are of paramount importance for the operational engineers that need to know in advance what problems (e.g. low pressures) can arise in a water distribution system during day to day activities or under abnormal operating conditions so that to be able to provide a quick and efficient response in order that the consumers not to suffer any detriment.

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