



# Managing Uncertainty in Operational Control of Water Distribution Systems

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## 1. Introduction

Operation of water distribution systems requires a variety of decisions to be made. There are system development decisions: where, when and what new elements of the distribution system need to be built. There are system management decisions concerning the regulatory measures such as water pricing principles, effluent standards, legislative measures etc.. There are operational decisions determining such things as water pumping schedules, reservoirs' operation and pressure control in a distribution network. These decisions are made at different time intervals and with very different time horizons in mind. The development planning spans a time horizon of 1-10 years and depends on forecasts of water demands, but it is also linked very strongly to regional planning through data on future social and economic conditions and future environmental conditions.

The management control, while it influences system development, concentrates on achieving optimum performance from the existing system. In this sense there is an interplay between system management regulations and operational decisions. The difference is in the possible or practical frequency of intervention: the prices, standards and other regulatory measures of a legislative nature cannot be changed too often and, in particular, they are not adjusted to actual hydraulic conditions and short term forecasts. The expected time horizon of management control is within the range 1-12 months.

As compared to this time scale, operational control decisions need to be made very frequently. For example, the decision to activate/deactivate pumps is made in response to measurements of reservoir levels or system

pressures collected on a minute-by-minute basis. Similarly the decision to isolate a burst pipe is made, without delay, as soon as the condition is identified. Consequently the time horizon for operator's control decisions spans from 1 minute to 24 hours.

This paper deals with the area of operational control which comprises of those decisions and actions that need to be varied in time, in response to current operating conditions and the actual state of the water distribution system. This is the area where the system dynamics play a dominant role, where the randomness of inputs cannot be neglected and where the physical and the system management constraints make the problem of control both difficult and challenging.

## **2. The objectives and constraints in operational control.**

Typical operational control decisions in water distribution systems concern reservoir(s) management with associated pump scheduling, and the control of the pressure profile throughout the system. The controlled system is dynamic, there are random disturbances and the control is subject to several inequality constraints.

In the most general terms, one can specify the objectives of operational control as the requirement to supply, in the most economical way, water of correct quality and quantity to all users, regardless of varying hydrological conditions and variability of consumptions. It means that one has to consider decision outcome variables such as flow rates in various pipes, water quality at various points in the system, reservoir levels, operating schedules of pumps etc..

Looking at the question of satisfying the control objective from the perspective of the system operator, we believe that the controls tend to be based on "averaged" rather than instantaneous states of the system (with an exception for reactions to emergencies which are clearly instantaneous).

Introduction of computer simulations into the operational control decision making, affords the evaluation of complex interactions between various control variables and departs from the conservative "averaged trajectory" control. The operator is given a facility to answer his "what-if" enquiries and to evaluate the consequences of the controls which reflect to a greater extent the current system state.

The latter implies however the need to face the problem of randomness of water consumptions and the need to consider uncertainty when making operational decisions.

### 3. Dealing with uncertainty in operational decision making.

Almost any operational decision focuses on uncertainty, [1]. Assume that the system operator can rely on his "what-if" (predictive) model of the system, that is the model which says: "if your decision will be  $\mathbf{u}$ , the present state of the system is  $\mathbf{x}_0$ , and the external influence (eg. variation of consumptions) is  $\mathbf{z}$ , then the outcome variable will be  $\mathbf{y}$ , as given by the formula"

$$\mathbf{y} = F(\mathbf{x}_0, \mathbf{u}, \mathbf{z}) \quad (1)$$

where  $\mathbf{y}, \mathbf{u}, \mathbf{z}$  are time functions and  $\mathbf{x}_0$  is a space-distributed state, formalised by decision theory. In principle, it should be possible to use with this model the utility theory developed for "risk conditions" (see for example [2] or [3]). For that purpose, however, we would have to assess: the subjective probabilities of the various external conditions and the risk factors associated with the outcomes which are predicted to take place under each external condition. All of that would have to be done on the basis of judgement of the individual decision maker when put into a similar situation. This may fail to be a practical approach as compared to a direct assessment of the outcomes.

The other approach developed in formal decision theory, known as "decision under uncertainty", makes a choice on the basis of the worst case, the best case, or some mix of both. The establishment of adverse scenarios still involves human judgement but it is made objective to a degree, by reflecting the hard constraints on the problem (see Bargiela and Hainsworth [7]). It seems that most experienced operators have a mental image of the optimistic and pessimistic scenarios and operate within these limits.

The question is to what extent will the mistrust towards the use of the expected value of the outcome of a decision be justified with respect to the operational decisions that are being made during normal system operation. It seems that two conditions are necessary for the expectation-based reasoning to be applicable to operational decision making: the situations have to repeat themselves many times within the considered time horizon, so that averaging can be effective, and moreover, the quantified benefit of control action must be a function of the average outcome of control decisions. In other words, "average" must be both meaningful technically and determinant of the judgement of the control system performance. If, for example, the control would be penalised for having caused a single pressure deficit at some point in the distribution network, it should not be based on the expected value of the state of the system.

In eq. (1),  $\mathbf{z}$  is a random-valued time function. Assume we know its appropriate probabilistic properties; then we could calculate the expected value of  $\mathbf{y}$ , that is

$$\bar{\mathbf{y}} = E[F(\mathbf{x}_0, \mathbf{u}, \mathbf{z})] \quad (2)$$

which tells the operator that "if the control decision is  $\mathbf{u}$ , then on average the outcome will be  $\bar{\mathbf{y}}$ ."

We could think of the outcome  $\bar{\mathbf{y}}$  as being the pressure profile trajectory (that is time function) integrated over a given time period. In which case one should consider to what extent, and when, will the expected value of  $\bar{\mathbf{y}}$  be meaningful for decisions.

There is, in this respect, a difference between the approach to operational control and to the management regulations. The management regulations are kept fixed over long periods of time; averaging is permissible and appropriate in that perspective, in particular if economic measures of system performance are considered.

#### 4. Scrutiny of information for operational decision making.

Let us continue to assume that the predictive model of the system is known, eq. (1). To make a decision at time  $t_0$  means to determine the time function  $\mathbf{u}$  for the control horizon  $[t_0, t_f]$ , denoted  $\mathbf{u}_{[t_0, t_f]}$ , such that the time trajectory  $\mathbf{y}_{[t_0, t_f]}$  of the outcomes exhibits certain properties desired by the decision maker. To be sure of what will be the resulting  $\mathbf{y}_{[t_0, t_f]}$ , we would have to know  $\mathbf{z}_{[t_0, t_f]}$  - the disturbance experienced by the system over the control horizon.

We would like to have several forecasts of  $\mathbf{z}$ , in order to be able to see the alternative outcomes of the proposed decision. From the operational control viewpoint, there is great value attached to the availability of a good forecast  $\mathbf{z}_{[t_0, t_f]}$ .

Now taking into account that not all control variables of the system belong to the operator; we denote the operator's control variables as  $\mathbf{u}^1$  and those of the water users as  $\mathbf{u}^2$ . The operator's control problem is to set values to  $\mathbf{u}^1$ . It is obvious that in the process of decision making the operator will need a prediction of  $\mathbf{u}^2_{[t_0, t_f]}$ . In general,  $\mathbf{u}^2$  cannot be considered as part of an external disturbance  $\mathbf{z}$ , since it is in some way dependent on  $\mathbf{u}^1$ , although it may also depend on some external disturbances,  $\mathbf{z}^2$ ; (for example weather conditions simultaneously influence consumer demands and the desirable operating pressure margins).

We shall therefore rewrite eq. (1) in the form

$$\mathbf{y} = F(\mathbf{x}_0, \mathbf{u}^1, \mathbf{u}^2(\mathbf{u}^1, \mathbf{z}^2), \mathbf{z}) \quad (3)$$

The notation  $\mathbf{u}^2(\mathbf{u}^1, \mathbf{z}^2)$  is an abbreviation for the "water user's model"

$$\mathbf{u}^2 = G(\mathbf{u}^1, \mathbf{z}^2) \quad (4)$$

Incidentally there is a likely pitfall when trying to calculate average values on the basis of eq. (3); one should not overlook the fact that  $\mathbf{z}^2$  and  $\mathbf{z}$  are correlated.

In summary, the necessary information for operational decision making is as follows:

- a model of the water system,
- the present value of the system state,  $\mathbf{x}_0$
- a forecast of external disturbances,  $\mathbf{z}$
- model (models) of water users,
- forecasts of external influence on the users,  $\mathbf{z}^2$ .

An experienced operator of a system would have all the models, implicitly, in his mind; he also makes the necessary forecasts, although he may never separate the concerns. In trying to improve operational decision making we will use formal (mathematical) rather than judgmental models and predictions. The degree to which we can assist the operator depends on how good system models can be built and how reliable predictions can be made. This, in turn, depends on *a priori* knowledge of the system as well as on the availability and accuracy of current data. Considerable research effort has been invested in modelling and forecasting for operational purposes.

#### 4.1 Network Model

The quality (accuracy) of this model will influence the quality (accuracy) of the simulation results. The refinement of the network model is an iterative process which starts with approximate engineering guesses to system parameters and progresses through the cycle of comparisons of simulation results with the actual measurements. The way the appropriate measurement set is assembled and the measurement discrepancies are translated into the corrections to model parameters gives rise to various model calibration techniques, [5],[14],[15],[16]. It must be recognised however, that whatever is the technique for refining the mathematical model, the accuracy of the model parameters reflects a compromise between the investment of calibration effort and the quality of subsequent simulations using this model.

#### 4.2 Forecasts of external disturbances

As the water distribution system is primarily driven by users' consumptions, the random variations of these constitute a disturbance to system operation. Early work on the predictions of consumer demands centered on predictions of aggregate consumptions (which frequently were

equivalenced to total system supply). A degree of success was reported by a number of researchers who modelled these aggregated consumptions using time series techniques [6],[12],[13], or categorisation and harmonics analysis [9],[11].

The issue of prediction of demands in individual nodes has been relatively less well researched. A step in this direction has been made by some water companies investigating night consumptions in small groups of nodes (zones) in order to localize excessive leakages. However, there have been no reports on explicit forecasts of external disturbances in individual nodes. An approach complementary to external disturbances modelling, based on the use of micro-events simulations, has been proposed in [8]. The statistical parameters of the simulated external disturbances have been shown to match quite well those of the observed random processes.

### **4.3 Water network model**

Some work has been done on the identification of the relationship between system pressure and leakages in the distribution networks, [5]. A similar study is needed to investigate the controllability of regular water users. This should also involve the identification and the measurement of disturbances affecting the users.

The subsequent step is to use the system models and forecasts of external disturbances in the context of formalised judgement when facing uncertainty. There are two difficult components of that task: the skill to assign values to various outcomes of a single control decision, so that one could choose the best control in a given future scenario; and the other skill, to be able to take into account that the conceivable scenarios are uncertain.

The first problem is that of ranking multi-attribute alternatives; the second problem is that of adding another dimension, that of uncertainty. The application of Knowledge Engineering techniques to the problem of automating operational decision making is a subject of current research by several researchers and, although it shows some promise, there have been few reports on successful industrial implementations.

## **5. Operational decision making mechanisms**

Operational control will probably always be done closed-loop; observation of actual system behaviour will be used to improve control decisions, which means that observations will be performed and used well before the end of the time horizon of control is reached. There exist two principal ways of using those observations: the state feedback decision rules and the so-called repetitive control structure.

### 5.1 The state-feedback rule

A simple state feedback decision rule, would look as follows. One would measure the instantaneous state  $\mathbf{x}(t)$  and devise controls  $\mathbf{u}(t)$  as a fixed function of the state

$$\mathbf{u}(t) = r(\mathbf{x}(t)) \quad (5)$$

In that form the rule does not take into account any forecast of the disturbance input  $\mathbf{z}$  to the system nor the actual prediction of the user behaviour  $\mathbf{u}^2$ , (compare eq (3)). In order to provide for optimality of control if the controlled system is time-varying, the rule itself must become time-varying too. So the rule should include a forecast  $\bar{\mathbf{z}}_{[t_0, t_f]}$  as an additional argument to become

$$\mathbf{u}(t) = r(\mathbf{x}(t), \bar{\mathbf{z}}_{[t_0, t_f]}) \quad (6)$$

The rules eqs. (5) and (6) are never "instantaneous", in the sense of being able to relate instantaneous controls to the instantaneous value of the state. Instead they are applied in a discrete-time version. For each period of time, one considers the state  $\mathbf{x}_i$  at the beginning of the period, the state  $\mathbf{x}_{i+1}$  at the end, the disturbances within the period  $\mathbf{z}_i$  and the controls  $\mathbf{u}_i$ . The operational decision is made at the beginning of the period, when the actual  $\mathbf{z}_i$  is unknown, therefore a forecast  $\bar{\mathbf{z}}_i$  is indispensable. The decision rule may be spelled out in terms of the desired state  $\mathbf{x}_{i+1}$

$$\mathbf{x}_{i+1} = r_x(\mathbf{x}_i, \bar{\mathbf{z}}_i) \quad (7)$$

or in terms of the controls

$$\mathbf{u}_i = r_u(\mathbf{x}_i, \bar{\mathbf{z}}_i) \quad (8)$$

One can see that the two approaches differ in their sensitivity to the uncertain  $\mathbf{z}_i$ . When rule (7) is used, we "insist on the state", that is the state  $\mathbf{x}_{i+1}$  is obtained at the cost of extra control which needs to compensate for  $\mathbf{z}_i \neq \bar{\mathbf{z}}_i$ .

When rule (8) is used, we "insist on control schedule", that is we accept that the desired state  $\mathbf{x}_{i+1}$  may not be obtained if  $\mathbf{z}_i \neq \bar{\mathbf{z}}_i$ . In practical cases a compromise between these two extreme approaches may be appropriate.

The actual form of decision rule (7) or (8) is established by parametric optimisation of a suitable performance index which, under a given decision rule, will depend on the disturbance  $\mathbf{z}$  and the parameters  $\mathbf{b}$ ,  $J=J(\mathbf{b}, \mathbf{z})$ . We should then maximize the expected value of  $J$ , that is solve the problem

$$\max_{\mathbf{b}} E[J(\mathbf{b}, \mathbf{z})] \quad (9)$$

## 5.2 The repetitive control

An alternative to the "fixed decision rule" described above is, the so called, "repetitive control" or "discrete feedback" algorithm. The basic principle is very simple: knowing, at time  $t_0$ , the initial state  $\mathbf{x}_0$  and a forecast of disturbance  $\bar{\mathbf{z}}_{[t_0, t_f]}$  one chooses a decision  $\mathbf{u}_{[t_0, t_f]}$  that provides, according to the model of the system, for a desired outcome  $\mathbf{y}_{[t_0, t_f]}$ . It is irrelevant for this control method to know how the control decision was arrived at. If the decision  $\mathbf{u}_{[t_0, t_f]}$  was applied to the system over the whole horizon  $[t_0, t_f]$  without any revision, we would have open-loop control. In the discrete feedback algorithm the model-based decision is implemented only over some period  $T$ , which is shorter (or much shorter) than the control horizon. At time  $t_0+T$  the model-based decision making process is repeated for the new initial state  $\mathbf{x}(t_0+T)$ , the new forecast  $\bar{\mathbf{z}}_{[t_0+T, t_f+T]}$  or even the new improved model. In that way the feedback information from the system is used, although not continuously.

Let us note that for a given system model and a given forecast of disturbances the fixed decision rule and the repetitive control method would deliver the same controls. Also, for the repetitive control, the adjustment of the system model is based on simulation of the system inclusive of control algorithms for a sufficiently rich population of external input sequences  $\mathbf{z}_i$ , that is in exactly the same way as we would adjust a decision rule.

The main difference and advantage of the repetitive control method lies in the fact that the process of determining the decision  $\mathbf{u}_i$  is explicit in the sense that the forecast, system model, constraints, preference ordering and risk attitude can all be made separate components of the decision making process. This feature is extremely important: it permits an interaction between the human operator and the computerised system. One can, for example, introduce man-made forecasts, change constraints and preferences, vary the risk assessment etc.. Moreover, since a system model is part of the algorithm, one can generate not only the current decision  $\mathbf{u}_i$ , but also future decisions and predict future system behaviour, which was envisaged by the algorithm when it recommended the decision  $\mathbf{u}_i$ .

As opposed to the repetitive control method, the "fixed decision rule" is an implicit device and permits no insight or intervention into the process - the only thing the operator can do is to obey or to ignore its recommendation.

## 6. Time horizons of operational control

Operational control is a dynamic control problem which means that any present decision has to take into account the effects that it will cause in the future. This raises the question of how distant the future has to be considered, that is, what should be the control horizon  $t_f$ . It is very important to note that whatever time horizon  $t_f$  will be chosen, the problem is of the kind where the final state of the system  $\mathbf{x}(t_f)$  is not free. The water system will continue to operate after any time  $t_f$  that we choose, and an adequate state of the system must be maintained to this effect. This sets a non-zero final state condition:

$$\mathbf{x}(t_f) = \mathbf{x}_d(t_f), \quad \text{or} \quad \mathbf{x}(t_f) \in \mathbf{X}_d(t_f)$$

where  $\mathbf{X}_d(t_f)$  is a set in the  $\mathbf{X}$ -space, of permissible states.

The problem of operational control horizon,  $t_f$ , does in fact reduce to the question how adequate are we able to set the desired final state  $\mathbf{x}_d(t_f)$  or the target set  $\mathbf{X}_d(t_f)$ . If the optimal value  $\mathbf{x}_d(t_f)$  calculated at  $t=0$  does not depend on the state of the system at time zero,  $\mathbf{x}(0)$ , nor on the current values of the disturbances which are observed at  $t=0$ , then the time horizon  $t_f$  is long enough. Note that in that case the value  $\mathbf{x}_d(t_f)$  will be set on the basis of long-term data only, such as the diurnal variation pattern. Practically speaking, if, in setting the desired state for mid-day tomorrow, we can do no better than to use a typical mid-day state of the system as a reference, with no respect to current state, then a day's horizon is all that is needed.

The statement that  $\mathbf{x}_d(t_f)$  calculated at  $t=0$  does not depend on  $\mathbf{x}(0)$  means that the horizon  $t_f$  is long enough from the viewpoint of the dynamics of the system. With small storage capacities in the system, we need a relatively short horizon; for example, if the morning reserve does not restrict setting the reserve that is required for mid-day, then from the system dynamics point of view we do not need more than 6 hours horizon.

Another factor to take into account in setting the time horizon is the forecasting lead of the disturbance. If, in the above example, we can forecast in the morning that, until noon, the consumptions are going to be different from the long term, diurnal forecast, then the operational controls should depart from the long term average settings. The forecasting lead ends at mid-day, if the morning forecast of the afternoon consumptions is no more accurate than the diurnal forecast.

In summarising, the operational control horizon should be as short as possible but no shorter than the forecasting lead and no shorter than the value of  $t_f$  dictated by the system dynamics.

## 7. Conclusions and acknowledgements

It is believed that the introduction of uncertainty processing techniques into current computerised telemetry systems deployed in the water industry, is an indispensable component of computer assisted operational control. While the deterministic simulations have demonstrated their benefits in a number of applications [4, 5, 10], the probabilistic mathematical models are needed to provide more realistic assessment of the outcomes of on-line control decisions [7]. Also, such models are seen as reflecting more naturally the operator's reasoning about the system.

It must be spelled out however that probabilistic system modelling does not remove the need for a human operator to close the control loop. This is particularly pronounced in 'non-standard' situations and demonstrates the operator's ability to make a choice in multi-objective situations and under uncertainty; difficult to formalise tasks.

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